Math Circle Problems Sampler

Believe it or not, problems 2,3,4,5,and 6 are all related to one another!

1 A census-taker knocks on a door, and asks the woman inside how many children she has and how old they are.

"I have three daughters, their ages are whole numbers, and the product of the ages is 36," says the mother.

"That's not enough information," responds the census-taker.

"I'd tell you the sum of their ages, but you'd still be stumped."

"I wish you'd tell me something more."

"Okay, my oldest daughter Annie likes dogs."

What are the ages of the three daughters?

2 What are the maximum number of "Friday the 13ths" that can occur in a normal 365-day year? What are the minimum number that must occur? (Recall that April, June, September and November each have 30 days, February has 28 days, and all the other months have 31 days.)

3 Two people take turns cutting up a rectangular chocolate bar which is 6×8 squares in size. You are allowed to cut the bar only along a division between the squares and your cut can be only a straight line. For example, you can turn the original bar into a 6×2 piece and a 6×6 piece, and this latter piece can be turned into a 1×6 piece and a 5×6 piece. The last player who can break the chocolate wins (and gets to eat the chocolate bar). Is there a winning strategy for the first or second player? What about the general case (the starting bar is $m \times n$)?

4 Three frogs are placed on three vertices of a square. Every minute, one frog leaps over another frog, in such a way that the "leapee" is at the midpoint of the line segment whose endpoints are the starting and ending position of the "leaper." Will a frog ever occupy the vertex of the square that was originally unoccupied?

5 Consider the following two-person game: You start with an $n \times m$ grid of graph paper. Players take turns coloring red one previously uncolored unit edge of the grid (including the boundary). The player who creates the first closed red path loses. Is there a winning strategy for one of the players?

6 Lockers in a row are numbered $1, 2, 3, \ldots, 1000$. At first, all the lockers are closed. A person walks by, and opens every other locker, starting with locker #2. Thus lockers $2, 4, 6, \ldots, 998, 1000$ are open. Another person walks by, and changes the "state" (i.e., closes a locker if it is open, opens a locker if it is closed) of every third locker, starting with locker #3. Then another person changes the state of every 4th locker, starting with #4, etc. This process continues until no more lockers can be altered. Which lockers will be closed?

7 I invite 10 couples to a party at my house. I ask everyone present, including my wife, how many people they shook hands with. It turns out that everyone shook hands with a different number of people. If we assume that no one shook hands with his or her partner, how many people did my wife shake hands with? (I did not ask myself any questions.)

For solutions, come to the San Francisco Math Circle!